CHAPTER 18
Bargaining Problems & Nash’s Solution
Bargaining: Creation & Division of Value

• Parties often negotiate over terms of engagement
• Example: Firm and worker negotiate over whether and how to hire/work together
• Example: Patient and dentist negotiate over treatment options, costs, and financing
• Bargaining *creates* value (dentist earns income, you fix or avoid pains) and *divides* value (fees, salary)
• The division depends on social norms, skills, starting positions
• Usually bargaining has a *default outcome* associated with failing to reach an agreement.
Representation of bargaining problems

- We will, in particular apps, detail all the information, skills, and options available to the two bargaining parties.
- For now, we view bargaining problems as a whole only abstractly, to appreciate John Nash’s insight.
- **Bargaining problem** (two players) is a pair \((V,d)\) with \(d \in V\):
  - \(V\) = set of 2-vectors of payoffs reachable by bargaining;
  - \(d\) = default/disagreement vector. Eg: \(V=\{(4,6),(2,2)\}\), \(d=(2,2)\).
- Assume **monetary transfers** are possible:
  - \(t\) = transfer from 2 to 1 (if negative, it means \(|t|\) from 1 to 2)
- Assume **transferable utility**: If \(z\) represents non-monetary agreement, then utilities can be written as
  \[
  u_1 = v_1(z) + t \quad \text{and} \quad u_2 = v_2(z) - t
  \]
Representation of bargaining problems

- **Monetary transfers**: \( t = \) transfer from 2 to 1
  
  **Transferable utility**: Utilities \( u_1 = v_1(z) + t \) & \( u_2 = v_2(z) - t \)

- Implication: If \((2,2)\) belongs in \( V \), so does \((2+t,2-t)\)
  
  If \((4,6)\) belongs in \( V \), so does \((4+t,6-t)\), eg \((6,4)\)

- The sum \( u_1 + u_2 \) is called the **joint value**. It equals \( v_1(z) + v_2(z) \)
  
  ... transfers affect value’s division, not creation

  Fixed joint value is any line sloped -1

- **Surplus value** = joint value – \( d_1 - d_2 \)

- A bargaining outcome \( v \in V \) is **efficient** if \( u_1 + u_2 \) is maximized. This def. agrees with prior one, given transferable utility
Example

- Rose a HS principal, Jerry an actor
- They jointly decide whether Jerry works at HS; if so, whether responsible for drama alone (z=0) or also for softball (z=1)
- Jerry $v_J(z) = 10,000 - 3,000z$ ... values teaching drama, dislikes softball’s evening hours
- Rose $v_R(z) = 40,000 + 5,000z$ ... values adding drama & softball
- Rose & Jerry on same side (opposite) as for drama (softball) but Rose values drama far more than does Jerry
- Joint value is $(10000-3000z) + (40000+5000z) = 50000 + 2000z$
  This is positive, and greater at $z=1$ than at $z=0$, despite Jerry. A monetary transfer can convince Jerry and reach efficiency.
- If $d_J=15000$ and $d_R=10000$, then surplus is $25000 + 2000z$. 

Bargaining solutions

- A bargaining solution is a function that assigns to any bargaining problem \((V,d)\) a “feasible” outcome \(f((V,d)) \in V\).

- Examples:
  - \(f = \text{Maximize surplus, give all to player 1}\)
  - \(f = \text{Maximize surplus, give all to player 2}\)
  - \(f = \text{Average surplus, split evenly}\)
  - \(f = \text{Give each the default payoff}\)
John Nash’s “standard bargaining solution”

- Give player $i$ a bargaining weight $\pi_i \geq 0$ (with $\pi_1 + \pi_2 = 1$) that represents power, skills, starting position, information, ...

- The standard bargaining solution:
  1. Calculate actions $x^*$ that maximize joint value – call it $v^*$
  2. Player $i$ gets welfare $d_i + \pi_i(v^* - d_1 - d_2)$
  3. To achieve this, use transfer $t$ that solves

\[
d_1 + \pi_1(v^* - d_1 - d_2) = v_1(x^*) + t
\]

Note, $d_2 + \pi_2(v^* - d_1 - d_2) = v_2(x^*) - t$ automatically is implied.

Basic idea: Each player’s welfare in excess of default outcome equals his share/weight of the joint surplus $\pi_i(v^* - d_1 - d_2)$

Thus if $\pi_J = 1/3$ & $\pi_R = 2/3$ then $u_J = 15K + 27K/3$ & $u_R = 10K + 2*27K/3$
Axiomatic derivation of bargaining solutions

• Latter seems pulled out of thin air, though reasonable

• Is it “compelling”? One way to make it so is to ask for acceptance of a few basic, primitive statements (1+1=2), then show how they imply the standard bargaining solution.

• John Nash derived his bargaining solution from three axioms. In doing so, he founded a new filed, of “axiomatic bargaining theory” … in which axioms are posited, and bargaining solutions derived.
I. Invariance axiom

If a given bp \((V,d)\) is changed to \((V',d')\) as follows

\[
V' := \{(a_1(v_1 - d_1), a_2(v_2 - d_2)) \mid v \in V\}
\]
\[
d' := (0,0)
\]

... by shifting \(d\) to origin & scaling payoffs by \(a_{1,2}\) ...

then the bs \(v'\) must so change from the original bs \(v^*\):

\[
v'_i = a_i(v^*_i - d_i)
\]
Invariance axiom

- Shift V so default touches origin $\rightarrow \star$ shifts in same direction, extent
- Halve player 2’s scale, so his feasible payoffs double $\rightarrow \star_2$ also doubles
- That is, player’s share of surplus (75%) preserved under shifts & scalings
Efficiency axiom

II. Efficiency axiom: For any bp \((V,d)\), the solution \(f(V,d)\) must be in the efficient frontier of \(V\) above default outcome.

- not efficient
- not above default
- along here OK
IIA

• An outcome is relevant if it is at least as good, for each player, as the default outcome (i.e., if \((v_1, v_2) \geq (d_1, d_2)\))

• Given two bp’s \( (V,0),(W,0) \), say V is a relevant-shrinkage of W iff every relevant outcome in V is also in W.

III. Independence of Irrelevant Alternatives

Fix a bp \( (W,(0,0)) \) and consider any relevant-shrinkage \( V \). Then \( f(V,0)=f(W,0) \) provided \( f(W,0)\in V \).

“Bargaining solution must not change, on removing irrelevant outcomes, so long as solution itself has not been removed.”
IIA axiom

contains irrelevant outcomes  solution immune to eliminating them
Nash derived, not assumed, his standard solution

• **Nash’s result.** Fix a bargaining solution. Then it satisfies axioms I,II,II $\Leftrightarrow$ it is the standard solution for some weights $\pi$

Why? See p.437-9 in Appendix D.

• Interpretation
  - Standard bargaining solution is justifiable with these axioms
  - No other bargaining solution satisfies these axioms
  - If you accept these axioms, you must accept standard b.s.
Hiring top programmers

• John (job applicant) and firm face a bargaining problem, with default outcome (120K,0) if hiring does not take place. John is the top expert, he can work elsewhere for at least $120K, and firm would have to hire the next best, far worse.

(a) John can program or manage | (b) John can get salary $t

• $u_J^{(\text{program})} = t + 30K$, $u_J^{(\text{manage})} = t - 40K$ (loves programming)

• $u_F^{(\text{program})} = 150K - t$, $u_F^{(\text{manage})} = 200K - t$ (prefers manager)

Surplus is 180K – 120K (if program) & 160K – 120K (if manage)

• Say bargaining weights $\pi_{J,F} = .75, .25$ (all know none like John)

Nash’s standard solution: John hired as programmer with $t = d_J + \pi_J (40K) - (v_J^{(\text{prog.})}) = 120K + 45K - 30K = 135K$

Salary is $135K$, firm gets net welfare of $0 + .25(60K) = 15K$