8. Imagine a game in which players 1 and 2 simultaneously and independently select A or B. If they both select A, then the game ends and the payoff vector is \((5, 5)\). If they both select B, then the game ends with the payoff vector \((-1, -1)\). If one of the players chooses A while the other selects B, then the game continues and the players are required simultaneously and independently to select positive numbers. After these decisions, the game ends and each player receives the payoff \((x_1 + x_2)/(1 + x_1 + x_2)\), where \(x_1\) is the positive number chosen by player 1 and \(x_2\) is the positive number chosen by player 2.

(a) Describe the strategy spaces of the players.
(b) Compute the Nash equilibria of this game.
(c) Determine the subgame perfect equilibria.

9. Consider the following two-player game. First, player 1 selects a real number \(x\), which must be greater than or equal to zero. Player 2 observes \(x\). Then, simultaneously and independently, player 1 selects a number \(y_1\) and player 2 selects a number \(y_2\), at which point the game ends. Player 1’s payoff is

\[u_1 = x_1(y_2 + x_1) - y_1 - \frac{x^2}{3}\]

and player 2’s payoff is

\[u_2 = -(y_1 - y_2)^2\]

Calculate and report the subgame perfect Nash equilibrium of this game.

10. This exercise will help you see that subgame perfection does not embody the notion of forward induction presented at the end of this chapter.

(a) Consider the game in Figure 15.5. Calculate and report the subgame perfect Nash equilibrium strategy profiles. Are all of these equilibrium strategy profiles consistent with iterated conditional dominance? If not, comment on the equilibrium belief of player 2 and how it differs from the belief identified in the iterated conditional dominance procedure.

(b) For the game described in Exercise 7, can you find a subgame perfect equilibrium that does not survive the iterated conditional dominance procedure?

11. Consider the following game.

```
 1 2 3 4 5 6
A C H I J K
B D E F G
```

(a) What values of \(p_1\) and \(p_2\) are needed to make \((C, C)\) a Nash equilibrium of the induced game?
(b) What values of \(p_1\) and \(p_2\) will induce play of \((C, C)\) and would arise in a subgame perfect equilibrium of the entire game (penalty announcements followed by the prisoners’ dilemma)? Explain.
(c) Compare the unilateral commitments described here with contracts (as developed in Chapter 13).

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(a) How many proper subgames does this game have?
(b) Solve the game by backward induction and report the strategy profile that results.
(c) Find the set of strategies that survive iterated conditional dominance. Compare these with the strategy you found for part (b).
(d) Compare the path through the tree that results from the strategy you found for part (b) with the paths that are consistent with iterated conditional dominance.

12. Suppose players 1 and 2 will play the following prisoners’ dilemma.

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>5.5</td>
<td>0.8</td>
</tr>
<tr>
<td>B</td>
<td>7.0</td>
<td>1.1</td>
</tr>
</tbody>
</table>

Prior to interacting in the prisoners’ dilemma, simultaneously each player \(i\) announces a binding penalty \(p_i\) that this player commits to pay the other player \(j\) in the event that player \(i\) defects and player \(j\) cooperates. Assume that these commitments are binding. Thus, after the announcements, the players effectively play the following induced game:

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>5.5 (-p_2)</td>
<td>8 - (p_2)</td>
</tr>
<tr>
<td>B</td>
<td>7 - (p_1), (p_2)</td>
<td>1, 1</td>
</tr>
</tbody>
</table>

(a) What values of \(p_1\) and \(p_2\) are needed to make \((C, C)\) a Nash equilibrium of the induced game?
(b) What values of \(p_1\) and \(p_2\) will induce play of \((C, C)\) and would arise in a subgame perfect equilibrium of the entire game (penalty announcements followed by the prisoners’ dilemma)? Explain.
(c) Compare the unilateral commitments described here with contracts (as developed in Chapter 13).

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