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Quantifying the Cost of Risk in Consumption

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Quantifying the Cost of Risk in Consumption*

Mario Tirelli and Sergio Turner

Abstract

Fixing a risky intertemporal, interagent consumption profile, its **total cost** is the total willingness to pay for smoothing everyone's consumption. It decomposes into a **micro cost** that captures the inefficiency in the cross-sectional distribution of total consumption, risky as it is, and a **macro cost** that captures the additional benefit of eliminating the risk in total consumption, once efficiently redistributed.

We consider the risk that a household experiences income mobility and the consequent consumption mobility. U.S. panel data estimates a consumption profile for which we compute the costs. The total cost is 9-18% of total initial consumption for CRRA parameters 1.25-3.5. Of this, 80-90% is the micro cost and only 10-20% is the macro cost. The magnitude of these results, and in particular the relative importance of the micro cost, is in line with previous empirical evidence.

Motivated by this evidence we develop the theory of micro cost. Moreover, because the micro cost does not admit a closed form, for general preferences, we lay out an approximation method.

KEYWORDS: inefficiency, risk in consumption, incomplete markets, willingness to pay, consumption mobility, social welfare, business cycle

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1 Introduction

Data show that individual consumption is variable both across time and, in the cross section, between individuals, or groups, with different demographic and socio-economic characteristics (e.g. age of birth, family characteristics, education, wealth, health and employment history, etc.). In the perspective of a single individual, this variability in future consumption configures a *risk*. Risk averse agents do not efficiently diversify risk in individual consumption because of market incompleteness and frictions, as well as because of limited redistributive schemes, such as social insurance or family and community arrangements.

Risk in individual consumption imposes a cost on individual welfare, measurable as the willingness to pay to achieve its efficient diversification. Thus risk in a profile of consumptions has a *total cost*, the sum of these individual costs. A natural goal is to design policies that reduce the total cost of risk in consumption, and toward it a natural step is to identify and quantify the components of the total cost. The trivial decomposition is one that considers every individual's willingness to pay; yet, this is unhelpful if there is much heterogeneity among individuals. An alternative, natural, decomposition is between *macro cost*, the component that is to blame on *macro risk*–the variability of aggregate consumption; and *micro cost*, the component that is to blame on *micro risk* –the cross-sectional variability of consumption. Doing so, the damage to welfare is identified in two public risks, macro and micro risk, rather than heterogeneous private risks. Which is most damaging to welfare, the macro risk or the micro risk? Presumably, answering this would help to evaluate and design policies that target the total cost.

In this paper we (i) define a measure of total cost and its decomposition in a micro and macro component, for any profile of risky individual consumptions, (ii) present an empirical application of these measures for the risk profile induced by income in the US, finding that for this profile the micro risk is up to two orders of magnitude costlier than the macro risk, and accordingly (iii) develop the theory of the "micro component" of total cost, and discuss measurement problems. We detail the main contributions, and explain how our approach relates to the literature, next.

The basic notion is willingness to pay. Given an individual profile of present and future consumption $x^h = \left(x_0^h, x_{\mathbf{1}}^h\right)$, the *willingness to pay* in the present for a change z^h in future consumption, $w_x^h(z^h)$, is the scalar w^h that makes the individual indifferent between the new bundle, $\left(x_0^h - w^h, x_{\mathbf{1}}^h + z^h\right)$, and the status quo, x^h, given an ordinal preference.[1] The *total willingness to pay* for a change z in

[1]In general, we restrict z^h to be greater than $-x_{\mathbf{1}}^h$, so as to rule out negative consumption bundles. Moreover, to avoid moral hazard, payment is timed with, not postponed after, the willingness to make it.

future consumption is naturally $W_x(z) := \Sigma w_x^h(z^h)$. This entails no assumptions of interpersonal comparability or cardinality of preferences. The **total cost of risk in a profile of consumptions** x, is the maximum total willingness to pay for replacing future aggregate consumption by its expectation, and redistributing aggregate consumption across individuals:

$$T_x := \max_z W_x(z) \tag{1}$$

such that $z := (..,z_t,...)$, and in each future date-event t, z_t satisfies $\Sigma_h(x_t^h + z_t^h) = \mathbb{E}[\Sigma_h x_t^h]$. The **micro component** of total cost is the maximum total willingness to pay for *reallocating* future aggregate consumption in each date-event while holding fixed aggregate consumption:

$$m_x := \max_z W_x(z) \;\; s.t. \; \Sigma z^h = 0 \tag{2}$$

Metaphorically, an insurer sells the economy a plan of *mutual* insurance, beyond the mutual insurance tacit in x, for the greatest possible price, m_x. We show that $m_x = 0$ if and only if x is Pareto efficient, so m_x is interpretable as a measure of the inefficiency of the cross-sectional distribution of Σx^h. The **macro component** M_x is defined residually, as the total cost minus the micro component $T_x - m_x$. Hence, the following decomposition is a tautology,

$$T_x = m_x + M_x \tag{3}$$

To better interpret the macro component, we note program (1) is the same as (2), except that the aggregate change Σz^h goes from being 0 to being $\mathbb{E}[\Sigma x_{\mathbf{1}}^h] - [\Sigma x_{\mathbf{1}}^h]$, i.e. from preserving to removing risk in future aggregate consumption. The difference between these programs' values, M_x, is thus interpretable as the total willingness to pay for removing risk in future aggregate consumption. Metaphorically, the insurer now sells the economy another plan, which insures that future resources available for consumption will be $\mathbb{E}\left(\Sigma x_{\mathbf{1}}^h\right)$ instead of $\Sigma x_{\mathbf{1}}^h$, extracting a price M_x in addition to the original m_x. Whether or not this is feasible, in practice, is not the issue at stake here. Remember that our goal is to measure how the macro and micro risks contribute to the total cost, which is indeed captured by (3) in terms of the macro and micro components.

Second, we estimate the total cost, and its macro and micro components, of a large and persistent risk faced by US individuals. It is the risk that individuals experience long run consumption mobility. We estimate the total cost of this risk by computing T_x, where the consumption distribution x is estimated from US panel data over fifteen years, 1984-1999. The total cost is $9-18\%$ of 1984 total consumption, for CRRA parameters $1.25-3.5$ (see Figure 1). Of this total cost, the micro component accounts for $80-90\%$ and the macro component for the reminder $10-20\%$. For given preferences, these estimates are only as precise as the estimate of the consumption distribution x, so we underscore both caveats and validity checks.

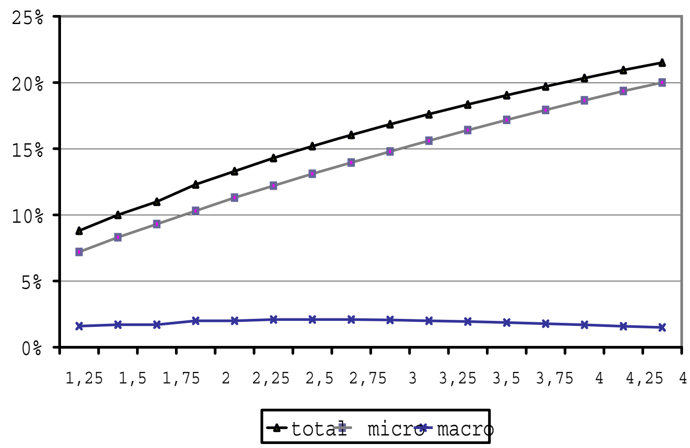

Figure 1: Cost components vs. risk aversion

These estimates strongly suggest that the micro risk damages social welfare far more than the macro risk, for a major risk faced by US individuals; and, as we argue latter, they are in line with previous studies on consumption.

Third, we study the theory of the micro component (2), for general preferences and consumption distributions. The motivation is twofold. Empirically, the micro component overwhelms the macro component for an important risk, according to the above estimates; theoretically, the micro component quantifies the inefficiency of the consumption distribution, as noted, complementing the qualitative literature on the inefficiency at equilibria of incomplete financial markets. We notice that the micro component does not admit a closed form, for general preferences, and layout an approximation method. In the process we highlight how a very intuitive approximation based on marginal rates of substitution may lead to large approximation errors; and explain why based on the fact that individual willingness to pay are decreasing for convex preferences. Then, in the context of our empirical application, we show that errors are indeed large.

Lastly, we make two extensions. One considers the possibility of transferring consumption across time, through a real or financial technology. There is a similar notion of total cost and decomposition, now with a third component that quantifies the inefficiency in the intertemporal distribution of total resources $\Sigma_h x_t^h$. The second extension allows the possibility of multiple goods in each date-event.

Attanasio and Davis (1996) is the first work that defines and quantifies the micro component of welfare cost. They ask whether individuals mutually insure, and if not, what the welfare cost of imperfect mutual insurance is. Tacit in this question is the recognition that this cost is distinct from the welfare cost, which presumes that not only individuals mutually insure their consumption, but society as well insures against macro shocks. First, they document a spectacular failure of consumption insurance between age/educational groups of individuals. Second, they define the cost of imperfect mutual insurance, but quite differently from our m_x. Specifically, for them it is the willingness to pay of a fictitious agent (whose welfare is a sum of agents' welfare), out of life-time total consumption; whereas for us it is the sum of willingness to pay of actual agents, out of current consumption. These concepts do not commute, of course. Definition (2) seems more useful from the standpoint of a policy maker who also wishes to have an explicit distribution of costs and benefits from the reallocation that fixes the inefficiency quantified by (2); whether this is thought to help policy design or merely to extract a payment by actual agents (versus a fictitious agent). Despite these conceptual differences, we latter argue that our empirical finding, correctly interpreted, is in line with theirs, both in absolute and relative magnitude.

The issue of measuring the welfare cost of consumption fluctuations has also been central in the debate on the welfare consequence of business-cycles. Lucas (1987) asks about the ex-ante willingness to pay of a representative agent to substitute the actual time series of US aggregate consumption with its linear trend. In our words his measure corresponds to the total cost T_x, provided the base reference is initial (rather than life-time) consumption. With multiple agents a comparison becomes problematic, and the conceptual difference between the two measurement methods emerges. We begin by noticing that the measure of the cost of business cycle and our macro differ. In the metaphor where the policy maker is about to remove the business cycle shocks, the allocation of aggregate consumption is typically not Pareto efficient, as modeled, because agents bear idiosyncratic income shocks against which they can only incompletely insure. By contrast, in the metaphor defining our notion of macro risk, the policy maker already has implemented the mutual insurance plan and made the allocation of aggregate consumption Pareto efficient; that is, the macro cost is merely the welfare cost of the policy maker not also removing this aggregate consumption inefficiency. Next, we observe that there is a more fundamental difference between the two approaches. In the essence, our measure of total cost and its decomposition are defined upon two notions of efficiency, in (1) and (2); that is they unambiguously quantify the lack of efficiency, according to those notions. Instead, the typical measure of the welfare cost of business-cycle quantifies the welfare gain from removing business-cycle risk. The difference emerges observing two aspects. First, there is no obvious way to eliminate this aggregate risk without affecting idiosyncratic risk, given that the two are in general statistically dependent.[2] This is important, since different elimination methods may lead to different results. Second, the fact that removing a risk produces positive welfare gains does not unambiguously signal that it would be efficient to target the removal. For example, removing wage variability may have perverse effects on labor supply and efficiency, in Heathcote, Storesletten and Violante (2008). So, the two approaches differ, and we believe ours is the one to follow to design policy interventions that target the total cost.

Finally, because our micro cost identifies a failure of risk sharing efficiency, this paper complements the literature on general equilibrium of incomplete financial markets (GEI). The GEI literature establishes that when some insurance markets are missing, almost all initial distributions of resources lead individuals to an inefficient equilibrium allocation. Moreover, agents are in general not even capable to exploit the existent insurance markets to hedge risk in economies with multiple contingent commodities, Stiglitz (1982), Geanakoplos and Polemarchakis (1986).[3] Although conceptually remarkable, these contributions are purely qualitative; in our language, they prove that the micro cost is robustly nonzero if some insurance markets are missing. Yet they do not quantify this cost. If the GEI methodology is good at detecting generic inefficiency, it fails to provide a good method to quantifying it. Specifically, it shows that for almost every GEI equilibrium there is a direction of reallocation (in the sense of directional derivative) that is Pareto improving–this direction is identified by the agents' marginal rates of substitution (MRS). Thus might the MRS quantify the size of such Pareto improvements? the answer is that the MRS provide a natural approximation (an upper bound) on m_x (theorem 1); an approximation which, unfortunately, may be quite crude for risk averse agents, due to the *law of diminishing willingness to pay* (proposition 2).

The paper is organized as follows. **Section 2** outlines the theory of the micro component. **Section 3** estimates the total cost of consumption mobility in US, and its micro-macro decomposition. **Section 4** presents our method for deriving approximate formulas for the micro component (2), and discuss its accuracy both analytically and empirically. Finally, **section 5** contains the two extensions. An **appendix** collects most of the proofs.

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[2]A popular strategy is the "integrating principle", proposed by Krusell and Smith (1999), which substitute each actual, individual income profile with its expectation conditional to the realization of his idiosyncratic shock. An alternative is Atkeson and Phelan's (1994) who suggest to remove business cycle risk by removing the cross correlation of individuals' income processes.

[3]Various notions of *constrained inefficiency* have been studied to corroborate and extend these seminal contributions; for example, Geanakoplos et al. (1990), Citanna, Kajii, and Villanacci (1998), Citanna, Polemarchakis, and Tirelli (2006), Tirelli (2003), Turner (2005). For further references, see also Magill and Quinzii's (1996) book.

2 Quantifying inefficiency–the cost of micro risk

Inefficiency is a qualitative notion–we seek to quantify it. The idea is in the spirit of Debreu's (1951) coefficient of resource utilization, but emphasizes the individual willingness to pay and the timing of payment.

Without loss of generality, let us restrict to a two-period economy with uncertainty. Assume $\succeq$ is a complete preference ordering on $\mathbb{R}^{1+S}_{++}$, whose points $(x_0, x_{\mathbf{1}})$ specify consumption of a sole good in the present 0 and in the future states of nature $1,\ldots,S$. Let x be a status quo consumption, and $z \gg -x_{\mathbf{1}}$ a change in future consumption. The **willingness to pay** for this change, in terms of current consumption, is by definition the supremum w on $\mathbb{R}$ such that

$$x + (-w, z) \succsim x \tag{4}$$

Fixing the status quo, this defines a function $w = w(z)$ bounded above by x_0, whose argument we occasionally omit.[4]

If preference is continuous[5] and Inada in current consumption (i.e. $x_0 - w(z) > 0$), then (4) holds with indifference at the willingness to pay $w(z)$. Further, if preference is increasing in current consumption,[6] a solution w of $x + (-w,z) \sim x$ is unique.

The timing of payment–out of current consumption, not future–matches the interpretation of preferences as being *ex ante* the realization of the state of nature. *Ex post*, the willingness to pay in the realized state s may be different, if naturally defined, such as with state-separable preferences.

We quantify inefficiency in terms of willingness to pay. Let the **economy** $(\succeq, x)$ specify for each household $h = 1,\ldots,H$ a preference, as above, and a status quo consumption; let $(w^h)_h$ be the implied willingness to pay functions.

Definition 1 *The* ***inefficiency*** *of* $(\succeq, x)$ *is the value* m_x *of*

$$\sup \Sigma w^h(z^h) \qquad s.t. \qquad x_{\mathbf{1}} + z \gg 0, \Sigma z^h = 0 \tag{5}$$

The measure is for ordinal preferences, denominated in current resources, and lies in $[0, \Sigma x^h_0]$.[7] It is society's supremum willingness to pay for an allocation

[4] The set of w such that (4) is bounded above by x_0; if nonempty, its supremum exists by completeness of $\mathbb{R}$. Nonemptiness holds if the preference obeys **0-desirability**: for every x in $\mathbb{R}^{S+1}_{++}$ and z in $\mathbb{R}^{S}_{++}$, there exists a scalar w such that $x + (-w,z) \succeq x$.

[5] Continuity means that $(y_0, y_{\mathbf{1}}) \succ x$ implies $(\tilde{y}_0, y_{\mathbf{1}}) \succ x$ for $\tilde{y}_0$ in some neighborhood of y_0.

[6] Increasing in current consumption means that $x \in \mathbb{R}^{1+S}_{++}, \varepsilon > 0 \Rightarrow x + (\varepsilon, 0) \succ x$.

[7] $z = 0$ is feasible for (4) and has $w^h(0) = 0$, so $m_x \geq \Sigma w^h(0) = 0$. Since $w^h(z^h) \leq x^h_0$ whenever $z^h \gg -x^h_{\mathbf{1}}$, the sum $\Sigma w^h(z^h) \leq \Sigma x^h_0$, showing $m_x \leq \Sigma x^h_0$.

that, with the payment, is just weakly Pareto improving. A solution z of this problem, if it exists, is an **optimal arbitrage**. An arbitrageur could elicit Σw^h in the present, without adding to future resources.

Deferring proofs to the appendix, we now define a computationally useful characterization of optimal arbitrage:

Proposition 1 *Suppose in addition preferences are transitive. Suppose z in $\mathbb{R}^{SH}$ is feasible for (5). Then it is a solution if and only if the allocation $\left(x^h + (-w^h, z^h)\right)_h$ defines a Pareto optimum.*

This implies a characterization of Pareto efficiency:

Corollary 1 *x is Pareto efficient if and only if $m_x = 0$.*

Occasionally, we express inefficiency as an index, with value at most 1, by quoting the total willingness to pay as a fraction of current total consumption $r_0 := \Sigma x_0^h$: $m_x = \phi r_0$. So current consumption $c_0^h := x_0^h - w^h$ satisfies $\Sigma c_0^h = \tilde{\phi} r_0$ with $\tilde{\phi} := 1 - \phi$, which is an index of *efficiency* whenever $\Sigma w^h = m_x$. A similar notion is Debreu's (1951) coefficient of resource utilization $\tilde{\phi}$, except that he requires current *and* future consumption to satisfy $\Sigma c^h = \tilde{\phi} r$. His notion refers to the fraction of aggregate consumption, *in every state and not just today*, to which society is willing to deprive itself and still be weakly Pareto better off. As noted, with ex ante preferences, a willingness to pay today may disappear in a future state, a moral hazard hidden in Debreu's (1951) timeless model. The timing of payments and the emphasis on individual willingness to pay are the key distinctions between Debreu's measure and ours.

Lucas (1987) focus on the measurement of the welfare cost of business cycle, asks about the willingness to pay for changing future aggregate consumption $r_{\mathbf{1}} := \Sigma x_{\mathbf{1}}^h$ to their expectation, eliminating its risk. Again, this question is unrelated to micro cost and to our measure, which by the constraint $\Sigma z^h = 0$ fixes future aggregate consumption, preserving aggregate risk.

We end this section with two important remarks. First, reallocations in problem (5) are state contingent. Were they constrained to arise from a particular policy– fiscal, monetary, financial–then problem (5) would measure "policy constrained inefficiency." This would be no greater than our measure, as the feasible set would be no greater; our measure is an upper bound on "policy constrained inefficiency" for any policy.

Second, the micro cost of risk in individual consumption m_x is elusive, even when a formula for willingness to pay is available. To illustrate this point, consider preference which are quasilinear in current consumption, $x_0 + u_{\mathbf{1}}(x_{\mathbf{1}})$; a case in which the measure of individual willingness to pay coincides with other well known ones such as consumer surplus, compensating and equivalence variation. By our previous discussion, if utilities are time separable and Inada in current consumption, the individual willingness to pay w solves $u_0(x_0 - w) + u_{\mathbf{1}}(x_{\mathbf{1}} + z) = u_0(x_0) + u_{\mathbf{1}}(x_{\mathbf{1}})$. Rearranging and using quasilinearity, $w = u_{\mathbf{1}}(x_{\mathbf{1}} + z) - u_{\mathbf{1}}(x_{\mathbf{1}})$. Then, in the aggregate, $\Sigma w^h = \Sigma u^h_{\mathbf{1}}(x^h_{\mathbf{1}} + z^h) - \Sigma u^h_{\mathbf{1}}(x^h_{\mathbf{1}})$, so the micro cost (5) is the value

$$m_x := \left[\sup_{y_{\mathbf{1}} \gg 0, \Sigma y^h_{\mathbf{1}} = r_{\mathbf{1}}} \Sigma u^h_{\mathbf{1}}(y^h_{\mathbf{1}}) \right] - \Sigma u^h_{\mathbf{1}}(x^h_{\mathbf{1}}) \tag{6}$$

using the change of variable $y^h_{\mathbf{1}} = x^h_{\mathbf{1}} + z^h$, provided the solution has $x^h_0 - (u_{\mathbf{1}}(x_{\mathbf{1}} + z) - u_{\mathbf{1}}(x_{\mathbf{1}})) > 0$. The micro cost of risk in individual consumption is just the failure to maximize "future social welfare" $\Sigma u^h_{\mathbf{1}}$. This difficulty to derive an explicit formula for m_x, may motivate a search for approximations. Although this is not necessary for applications, as shown in the next section, we derive and discuss approximation methods in section 4.

3 Estimating the cost of inefficiency in US

Data provide a good accounting of consumption realizations, but clearly cannot identify unambiguously the state-space of an economy. Hence, any estimate of the welfare cost of risk in individual consumption depends, of course, on the particular risk and, using our terminology, on the emphasis placed on a particular risk component, whether micro or macro, in explaining the total welfare cost of risk in consumption. Our main goal in this section is to provide an estimate of the relative importance of micro risk, in the sense of m_x.

We tailor our application so as to emphasize a particular risk–that of US income mobility and the consequent consumption mobility. Empirical evidence shows that the risk of income mobility is large and persistent. Hence, it is a risk in income that is only partially insurable, causing consumption mobility. Indeed, Fisher and Johnson (2006) notice that, for the period 1984-1999, US income and consumption mobility are relatively closed;[8] and this despite the fact that, in the same fifteen years, a large increase in income inequality has not been followed by an equivalent increase in consumption inequality.[9] Thus, risk in our analysis is the consequence of the fact that individuals' income is subject to "shocks" over a long

[8]This is in line with the results in Jappelli and Pistaferri (2006) on Italian data.

[9]The dynamics of income vs. consumption inequality is well documented and explained, among others, in Krueger and Perri (2005).

time horizon; and these shocks cause individuals to change their position in the cross-sectional income and consumption distribution.

We keep our application simple by letting aside the analysis on how risk in income mobility translates into risk in consumption mobility, and directly focus on consumption risk and its micro cost.[10]

To grasp the essence of our construction, consider a two period economy of which we have a panel of individual consumption. Let (x_0, x_1) be the two cross sectional consumption distributions by quintiles of a panel of individuals. In the initial time period ($t = 0$), a sample of five individuals is drawn at random. Based on their individual consumption, each of them is assigned to an equinumerous quintile of the consumption distribution of the population, x_0. We treat each individual as representative of a quintile of x_0, by assigning him the average consumption of the quintile. The state space is defined by all the independent quintile transitions of the five sampled individuals, from x_0 to x_1, and the associated probability distribution is represented by a 5×5 -transition probability matrix. This state space allows to represent both aggregate (or "macro") and idiosyncratic risk. Loosely speaking, aggregate risk results form the fact that in a random sample multiple representative individuals may transition to the same quintile in $t = 1$ with some positive probability. Extreme cases, respectively with a representation of the lowest and highest aggregate consumption, being those in which all individuals achieve the same level of individual consumption corresponding to the first or the last quintile of x_1. Obviously, once a state in $t = 1$ realizes, the resulting consumption distribution in that state will be expressed in equinumerous consumption quintiles. Whether or not states involving multiple transitions are likely to occur depends on the associated probabilities. To sum up, our representation of uncertainty defines "macro shocks" as a function of the profile of individual shocks, not the other way around, and uses estimates of the probabilities of transitioning across consumption quintiles within a fifteen years period. This construction extends to any finite number of periods and individuals, giving rise to a date-event tree representation.

Finally, observe that the choice of working with five representative agents and two periods is computationally convenient; in this context the state space already results in $S = 5^5$ states. We interpret the two-period transitions, and the corresponding risk in consumption as the consequence of the inability of agents to fully insure against persistent income shocks in the intermediate periods. The final outcome in terms of individual consumption and inefficiency, as supposed to the period-by-period description, is what we focus on in our simple application.

[10] This is the strategy adopted in Attanasio and Davis (1996).

3.1 Data on income and consumption mobility

Fisher and Johnson (2006) estimate two tables of transition probabilities from 1984 quintiles to 1999 quintiles, respectively for disposable income and consumption expenditure (Table 1.1 and 1.2 below).[11] The tables are computed based on a synthetic panel data constructed imputing consumption data from the Consumer Expenditure Survey (CEX) to the households identified in the Panel Study on Income Dynamics (PSID).[12]

Transition probabilities
Income

↱	*1*	*2*	*3*	*4*	*5*
1	*0.50*	*0.26*	*0.14*	*0.08*	*0.03*
2	*0.24*	*0.29*	*0.22*	*0.18*	*0.08*
3	*0.12*	*0.21*	*0.23*	*0.27*	*0.17*
4	*0.08*	*0.16*	*0.23*	*0.26*	*0.27*
5	*0.06*	*0.11*	*0.17*	*0.21*	*0.45*

1.1

Transition probabilities
Consumption

↱	*1*	*2*	*3*	*4*	*5*
1	*0.51*	*0.26*	*0.13*	*0.06*	*0.04*
2	*0.24*	*0.27*	*0.24*	*0.18*	*0.07*
3	*0.13*	*0.20*	*0.22*	*0.25*	*0.19*
4	*0.07*	*0.19*	*0.25*	*0.26*	*0.23*
5	*0.05*	*0.09*	*0.15*	*0.25*	*0.46*

1.2

Table 1

The main diagonals of Table 1 report the proportion of individuals that remain in their respective, initial quintile; 1 being the proportion of those in the bottom of the distribution. For example, the first element in Table 1.1 indicates that 49.8% of individuals were in the bottom quintile of income in both 1984 and 1999. More than the 70% of individuals of the three middle quintiles experience a transition, while only approximately the 50% of the top and bottom quintiles do. The percentage of individuals who either remain in their quintile or move to the next (lower or higher) one, respectively, are $75\%, 75\%, 71\%, 87\%, 66\%$ ($76\%, 75\%, 68\%, 92\%, 71\%$ for Table 1.2), going from the poorest to the richest; hence, the complements to 100 of these figures indicate the probability of experiencing a sharp (up to 3 quintiles) change in consumption. Comparing the two tables reveals a similar pattern in the individual consumption and income transition. Overall, in the fifteen years considered, 65.3% of individuals move across the income distribution while 65.5% move across the consumption distribution; a difference

[11] Some rows and columns may not sum to one due to rounding.

[12] This well established methodology (see references in Fisher and Johnson (2006)) allows to obtain a better representation of household consumption by integrating PSID data, which are mainly on food and housing, with the cross-sectional data regarding other non-durable and durable consumption in the CEX.

that is statistically insignificant, according to Fisher and Johnson (2006). We interpret this as an evidence that long-run income mobility translates into consumption mobility.

To configure a risk in consumption, we consider CEX data on average expenditure, opportunely corrected. First, we correct data for family size to obtain measures for single individuals from household data.[13] Second, we express data in real terms using the US GDP deflator. Table 2 reports summary statistics of average real expenditure, with the data corrected for the family size in parenthesis. The standard deviation of the cross-sectional distribution by quintiles of the log consumption is 0.19 and 0.20, respectively, in 1984 and 1999. The growth factor indicates the aggregate, real consumption growth is 17.6% in the period 1984-1999. Finally, we notice that the percentage share of consumption across quintiles, with respect to total consumption, is $10\%, 14\%, 17\%, 23\%, 35\%$ in 1999, ($11\%, 14\%, 19\%, 21\%, 34\%$ in 1984). Therefore, a shift of exactly two quintiles, over the period, results in a 7% to 18% change of the individual share of total consumption in 1999.

Expenditure statistics - price 1999

	Mean	*St.Dev*	*St.Dev - logs*	*Growth factor*
1984	*33257 (20796)*	*18034 (9248)*	*0.227 (0.187)*	
1999	*39126 (24043)*	*22788 (11760)*	*0.249 (0.204)*	*1.176 (1.156)*

Table 2

3.2 Translation of data to model

Time Period 0 is 1984, period 1 is 1999.

Individuals They are five representatives, sampled one from each 1984 income quintile. We deal with this sample instead of the whole population to keep the state space manageable for numerical computation.

Preferences Individuals differ only in their patience parameters $\delta^h > 0$, otherwise having a common time separable, von Neumann-Morgenstern preference $v(x_0) + \delta^h \Sigma \pi_s v(x_s)$. The felicity $v(c) = \frac{1}{1-\beta} c^{1-\beta}$ has a CRRA parameter $\beta \in [1.25, 5]$, and π is the above state probability. We assume the annualized patience parameters $\delta^{\frac{1}{15}}$ to be $(0.92, 0.94, 0.96, 0.98, 1)$, increasing with first period consumption.[14]

[13]Following Fisher and Johnson (2006) we obtain an equivalent measure dividing household data with respect to the square root of family size.

[14]Qualitatively this profile is in line with standard applications. Quantitatively, the results we are going to present have been tested to be robust to absolute and relative shifts of these parameters.

States They are all the independent quintile transitions of the sampled households, totaling $S = 5^5$ states. Thus, $s = 55111$ is the state where the two poorest representatives become rich and the three richest become poor. In a random sample, multiple individuals can transition to the same 1999 quintile. As we noticed above, events in which multiple transitions occur result in a different future aggregate income.

State probabilities and distributions They are defined by the product rule for independent events, reflecting that individuals are a random sample, using Table 1.2 of consumption transitions. Thus, for example, the probability of state 55111 is $\pi_{55111} = (.044)(.072)(.129)(.074)(.047) \approx 8 \cdot 10^{-5}$, i.e. state 55111 is nearly impossible.

So Table 1.2 and consumption data define consumption distributions. To get an idea of how these distributions relate to the actual 1999 data, we explore corrected series in 1999 prices. At the individual level, agents in 1984, face a distribution of 1999 consumption with means and standard deviations reported in Table 3. In the first column of the table, in parenthesis, we also present the actual, corrected, expenditure of an individual ending up in the same quintile in 1999. We notice, that less mobile individuals (top and bottom quintile) have an expected consumption which is closer to their realized consumption in the event they remain in the same quintile of 1984. Individuals in 1984 face a total risk (standard deviation of 1999 consumption) which is moderately increasing in the initial quintile position, and exceeds the ex-post, cross-sectional standard deviation by roughly $50-100\%$. The expected growth rate of the economy exactly matches the actual. Finally, our assumptions on the state space produces a fairly high aggregate risk, the estimated standard deviation of log-aggregate consumption is 0.81;[15] something we expect to inflate the macro cost.

Individual expenditure statistics - price 1999

1984 quintiles	*Mean*	*St.Dev*	*St.Dev - logs*
1	*13119 (12485)*	*14001*	*0.317*
2	*20018 (16747)*	*16305*	*0.347*
3	*28792 (20889)*	*20910*	*0.389*
4	*31713 (27489)*	*21268*	*0.375*
5	*43942 (42606)*	*23084*	*0.380*

Table 3

[15]As a term of comparison, Lucas (2003) estimates the standard deviation of the log of real, per capita consumption to be 0.032 in the US, by measuring its actual standard deviation about a linear trend for the period 1947-2001.

3.3 Computations

We compute the micro cost of US consumption mobility by having Mathematica solve problems (1) and (2) for the model economy $(\succeq, x)$ just specified.[16] Estimates of cost as a percentage of 1984 total consumption, $\Sigma x_0^h =: r_0$, are

Cost components and risk aversion

β	*1.25*	*1.50*	*1.75*	*2.0*	*2.25*	*2.50*	*2.75*	*3.0*	*3.25*	*3.50*
$\hat{T}_x/r_0$	*8.8*	*10.0*	*11.0*	*12.3*	*13.3*	*14.3*	*15.2*	*16.1*	*16.9*	*17.6*
$\hat{m}_x/r_0$	*7.2*	*8.3*	*9.3*	*10.3*	*11.3*	*12.2*	*13.1*	*14.0*	*14.8*	*15.6*
$\hat{M}_x/r_0$	*1.6*	*1.7*	*1.7*	*2.0*	*2.0*	*2.1*	*2.1*	*2.1*	*2.1*	*2.0*

Table 4

This table indicates the estimated total cost is $9-18\%$ of 1984 total consumption, for CRRA parameters β in a range $1.25-3.5$. Moreover, of this total cost, the micro component accounts for $80-90\%$ and the macro component for just $10-20\%$ (see Figure 1 for a graphical representation).

How are we to interpret these figures about the model economy, which is not the whole economy but merely a sample, one individual from each quintile? Reallocations at the scale of the sample can be replicated to the whole economy, since quintiles are equinumerous. (Of course, many reallocations in the whole economy do not project to the sample). Thus reallocations in our model economy represent a subset of the reallocations in the whole economy. Since definition 2 involves a maximization over all reallocations, the inefficiency of the whole economy is at least the estimates in Table 4.

How are the figures compared with those in previous studies? If any comparison is made, one has to consider that our inefficiency measures are: $i)$ expressed as a fraction of initial, total consumption, $ii)$ rooted in the concept of individuals' willingness to pay. The closest to our analysis is Attanasio and Davis' (1996), who estimate the micro cost of eliminating consumption risk between age/educational groups to be around 2.7% of 10-years consumption, for a CRRA of 2 (see their Table 6); their macro cost is less than 0.1%. They interpret the first figure by saying that an individual with an annual real consumption expenditure of 20000$ would pay 540$ per year; that is approximately a (10 years-)present value of 4948$ (discounting at an annual 2% rate of interest). This is two payments of approximately 2230$ in the initial and tenth year. Instead, a similar agent in our economy,

[16] The code is available on request.

a representative of quintile three, would only pay 1381$ in the first year for an efficient redistribution in 15 years, for a CRRA of 2. This difference is a natural consequence of the *law of decreasing willingness to pay*, a general property of our measure we will discuss in the next section (see proposition 2). The *law* says that individual willingness to pay is concave, for convex preferences, implying that an agent whose payments are diluted over a longer period would be willing to pay a larger total amount. There is an additional aspect, related to ii), that, if considered, would reduce the difference between their estimates of inefficiency and ours. Attanasio and Davis measure the micro cost as the willingness to pay of a fictitious agent (whose welfare is the sum of agents' welfare), instead of the aggregate willingness to pay of many agents, as we do. To see how this may influence the results, we notice that estimating the micro cost with their measure would approximately double it: 22.2%, instead of 10.3%, of 1984 total consumption, for a CRRA of 2. Indeed, this qualitative result holds in general, except for the case in which agents have quasilinear preferences in current consumption.[17]

To sum up, even for somewhat different risk profiles,[18] our results, correctly interpreted, are in line with those in Attanasio and Davis' (1996). Yet, there is an important advantage in using our measure, which is again a consequence of item *ii*). Since our micro cost aggregates individual willingness to pay, we can keep track of the distributive gains and losses from inefficiency. For a CRRA of 2.5, the willingness to pay individual-by-individual to substitute actual consumption to the micro efficient are,

[17]This can be simply illustrated comparing the two programming problems defining the micro cost according to the two approaches. m_x/r_0 is the value of the problem,

$$max_y\left(1-\frac{\Sigma y_0^h}{r_0}\right) \text{ such that } (I)\; u(y^h)\geq u^h(x^h) \text{ for all h, } (II)\; \Sigma y_{\mathbf{1}}^h\leq \Sigma x_{\mathbf{1}}^h. \qquad \text{(P)}$$

Redefining the micro cost in the spirit of Attanasio and Davis (1996), with reference to a fictitious agent (whose welfare is a unweighted sum of agents' welfare), implies reducing the H constraints in (I) to a single constraint that aggregates the H. Clearly this modified problem has a constrained set that contains that of problem (P).

[18]To get a sense of how different is our economy form theirs, we just notice how they build their risk profile. First, they estimate actual consumption x, and compute the micro efficient consumption allocation (corresponding to perfect between-groups insurance) y. Second, they introduce a risk in consumption, assuming that, in every date t, individual consumption distribution is a two-point, uniform, taking either the actual value x_t or that value plus the gain from switching to the efficient, $x_t+(y_t-x_t)$.

Willingness to pay *(CRRA=2.5, price 1999)*

1984 quintiles	*w.t.p*	*w.t.p. (% 1984 CEX)*
1	*-270.60*	*-2.38*
2	*288.32*	*1.94*
3	*1731.39*	*8.65*
4	*2400.36*	*10.78*
5	*8531.82*	*24.06*
Micro cost	*12681.3*	*12.20*

Table 5

This suggests that even if the aggregate willingness to pay is high and positive, the individual may be very different and even negative, as it is for quintile 1. So, interpreting the willingness to pay of Attanasio and Davis' fictitious agent, as representative of the typical agent in the economy, may be very misleading.

Much more problematic, both conceptually and empirically, is to relate our figures with those in the large literature on the welfare cost of business cycle. Conceptually, as explained in the introduction, this cost is not equivalent to our macro cost. Empirically, the literature on the welfare cost of business cycle produced a fairly wide range of estimates: roughly comparable ones go from Lucas' (1987) 0.1% of lifetime consumption, to Krebs' (2007) $4-8\%$. What seams pivotal for larger estimates is whether shocks on individual income are modeled as persistent. In reviewing this literature, Lucas (2003) concludes that the only routes to larger estimates rely on (a) assumptions that convert small, transient aggregate shocks into larger, persistent (effectively uninsurable) individual shocks; Krebs (2003, 2007) and Storesletten, Telmer and Yaron (2001),[19] or on (b) there being long term risk in the representative agent's consumption, Alvarez and Jermann (2000), or on (c) the poorest being borrowing constrained and remaining poor along the path to a steady state; Krusell and Smith (1999), or on (d) an extremely high degree of relative risk aversion. Indeed, throughout this list there is always a risk that is highly persistent or permanent (including the risk of drawing a high risk aversion coefficient at birth). The idea that it is shock persistency to drive risk sharing inefficiency is well understood in theory and confirmed by a much wider empirical evidence. As for the theory part, in economies with incomplete insurance markets or market frictions (e.g. solvency constraints), persistent shocks explain the failure of the *permanent income hypothesis*.[20] As for the empirical counterpart, for example, contributions

[19]Macro crashes, although rare events, are also persistent shocks which weigh heavily on welfare– see Barro (2009), Chaterjee and Corbae (2007), and Salyer (2007).

[20]The permanent income hypothesis holds only under very restrictive assumptions, such as shocks which are transitory, assume iid, and agents which can smooth consumption by freely building and

go from those testing the mutual insurance hypothesis (e.g. Cochrane (1991)), to analysis on how income inequality leads to consumption inequality (e.g. Krueger and Perri (2006), Blundell et. al. (2008)), and to studies on the welfare cost of labor market risk (beside Attanasio and Davis (1996), Heathcote et. al (2008)).

4 Approximate measurement

Since the micro cost does not in general have an explicit formula, it may be helpful to analyze approximation methods. In doing so we generalize the idea that the micro cost of risk in individual consumption is just the failure to maximize "future social welfare" $\Sigma u_{\mathbf{1}}^h$; something we showed to be exact for quasi-linear preferences at the end of section 2.

We propose an approximation method of inefficiency m_x based on two-steps:

- derive an upper bound on willingness to pay, $w^h(z^h) \leq \hat{w}^h(z^h)$
- compute the value of the relaxed problem (or an upper bound thereof)

$$\sup_{x_{\mathbf{1}}+z \gg 0, \Sigma z^h = 0} \Sigma \hat{w}^h(z^h) \tag{7}$$

on substituting in the definition of m_x the upper bound of step one.

Clearly,

Principle 1 *Inefficiency is bounded above by (7).*

The first step is essentially a Taylor approximation of the equation characterizing willingness to pay, $u_0(x_0) - u_0(x_0 - w) = \Delta(z)$. Care is needed that this approximation is an *upper bound*. The method yields three approximations: linear, discrete, and quadratic. Henceforth, numerical computation is absent except to probe their accuracy in the economy presented in section 3.

drawing on a buffer asset (Bewley (1980, 1987)). Levine and Zame (1999, 2002) extended Bewley's result to economies with shocks which are recurrent Markov, implying also aggregate risk. They prove that the total cost of risk in consumption becomes negligible at equilibrium as infinitely lived agents become patient, provided they can borrow in limited amount, and can also trade an asset indexed on aggregate risk. In Levine and Zame (1999) they allow for shocks to be recurrent Markov, with aggregate risk. Then, the permanent income hypothesis holds for patient agents provided they can also trade "options on the social endowments".

4.1 Approximation by marginal rates of substitution

Marginal rates of substitution are appealing. They detect inefficiency both theoretically, that households' marginal rates of substitution are not all equal; and practically, that small Pareto improving reallocations solve some linear system. Being so good at detecting inefficiency, they should be good at quantifying it. Unfortunately, as we argue next, marginal rates of substitution (1) always overstate inefficiency, because they (2) ignore a law of diminishing willingness to pay, and (3) tag on a quantitatively gross error, even in reasonable cases.

Let $\widehat{w} = \nabla z$, where ∇ is the (intertemporal) **marginal rates of substitution (MRS)**, defined by the requirement that $(1, \nabla)$ be normal to the smooth indifference set through x. It turns out, this fulfills step one in the method:

Lemma 1 (linear approximation of willingness to pay) *Suppose the preference is reflexive and convex, and its indifference set through $x \gg 0$ is differentiable. The willingness to pay for z is at most*

$$w \leq \nabla z \tag{8}$$

By principle 1, inefficiency is bounded above by $* = \sup_{x_{\mathbf{1}}+z \gg 0, \Sigma z^h = 0} \Sigma \nabla^h z^h$, which involves the object

$$\triangledown^* := (\max_h \triangledown^h_s)_s \tag{9}$$

It is the stochastic maximum MRS over all households, an index of the most deprived. The solution of $*$ has all households fully donating their future consumption to the "most deprived":

Theorem 1 (linear approximation of inefficiency) *Suppose as in lemma 1. Then inefficiency is bounded above as*

$$m_x \leq \Sigma \left(\triangledown^* - \triangledown^h \right) x^h_{\mathbf{1}} := L_x \tag{10}$$

There is a reason that linearization overstates willingness to pay. Given a direction $z \in \mathbb{R}^S$ of change in future consumption, change $z(t) := tz$ is parameterized by its "size" $0 \leq t \leq 1$. How does the willingness to pay $w(z(t))$ for a change depends on its size?

Proposition 2 (law of diminishing willingness to pay) *Suppose the preference is increasing in current consumption and convex. Then $w(z(t))$ is concave. Further, $w(z(t)) \geq t w(z)$ for all $0 \leq t \leq 1$.*

The crudeness of marginal rates of substitution in approximating inefficiency is apparent in the economy of section 3. For a CRRA parameter of 2.5), the inefficiency is $m_x = .122 \cdot r_0$ and the linear approximation is over fourfold, $\Sigma\nabla^h z^h = .5 \cdot r_0 \leq L_x$.

This crudeness is present in the willingness to pay as well. Let us take the preference and status quo consumption to be that of the richest quintile in date 0, and the change z^5 in future consumption to be, say, that associated with the optimal arbitrage. The willingness to pay is $w^5 = 10677$ and the linear approximation (8) is nearly fourfold, $\nabla^5 z^5 = 41546$. The culprit of this crudeness is the law of diminishing willingness to pay[21], as illustrated by plotting $w^5(z(t))$ in Figure 2.

In sum, marginal rates of substitution are useful to detect inefficiency, but inept to quantify it. Granted, they are computationally much simpler than the inefficiency measure m_x. But so are the following approximations, sharper in general and accurate in the calibrated economy.

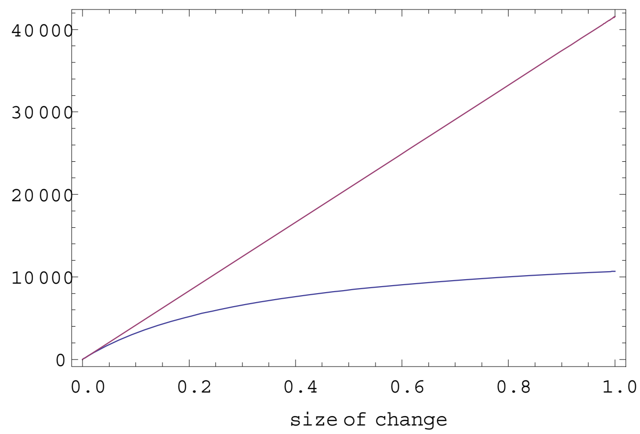

Figure 2: Linear approximation by MRS

4.2 Sharper approximations

We derive two approximations of inefficiency. Compared to the linear one based on MRS, they are sharper in general and dramatically so when tested on our data.

[21] Alvarez and Jermann (2004) linearize information about asset prices to answer Lucas' (1987) question. They find estimates far exceeding Lucas' (who does not linearize), but are mute on whether this excess owes to the linearization itself. They state an analogue of proposition 2.

Most surprisingly, inside them appears notion classically linked to efficiency, social welfare maximized subject to resource constraints:

$$F^* := \sup_{y_{\mathbf{1}} \gg 0, \Sigma y^h_{\mathbf{1}} = r_{\mathbf{1}}} \Sigma \mu^h u^h_{\mathbf{1}}(y^h_{\mathbf{1}}) \tag{11}$$

Here **future social welfare** $F : \mathbb{R}^{SH}_+ \to \mathbb{R}, F(y_{\mathbf{1}}) := \Sigma \mu^h u^h_{\mathbf{1}}(y^h_{\mathbf{1}})$ has the weights $\mu^h := \frac{1}{u^{h\prime}_0(x^h_0)}$, and **future resources** $r_{\mathbf{1}} := \Sigma x^h_{\mathbf{1}}$ are the status quo's. Preferences here admit a time separable representation $u_0 + u_{\mathbf{1}}$.

One application of the method results in

Theorem 2 (discrete approximation of inefficiency) *Suppose utilities for current consumption satisfy* $u'_0 > 0 \geq u''_0$. *Then the micro cost associated to* x *is at most the failure of this allocation to maximize future social welfare:*

$$m_x \leq F^* - F(x_{\mathbf{1}}) := D_x \tag{12}$$

Another application of the method results in involving the **total risk tolerance** $\overline{T}_0 := \Sigma T^h_0$, where $T_0 := -u'_0(x_0)/u''_0(x_0)$.

Theorem 3 (quadratic approximation of inefficiency) *Suppose utilities for current consumption satisfy* $u'_0 > 0, u'''_0 \geq 0 > u''_0$.[22] *Suppose also every individual has a nonnegative willingness to pay for the optimal arbitrage. Then the allocation's inefficiency is at most*

$$m_x \leq \sqrt{\overline{T}^2_0 + 2\overline{T}_0 \cdot [F^* - F(x_{\mathbf{1}})]} - \overline{T}_0 := Q_x \tag{13}$$

This quadratic approximation Q_x *is a correction of the discrete one* D_x : $Q_x = \sqrt{\overline{T}^2_0 + 2\overline{T}_0 \cdot D_x} - \overline{T}_0$

Observe that both approximations, discrete and quadratic, are explicit up to F^*, which to compute requires specifying future welfare $u_{\mathbf{1}}$, as illustrated below. Moreover, they are precise at Pareto efficiency. To see why, we recall the classical result that a Pareto efficient interior allocation solves $\sup_{y \gg 0, \Sigma y^h = r} \Sigma \mu^h u^h(y^h)$, which thanks to time separable utilities $u = u_0 + u_{\mathbf{1}}$ in turn implies $x_{\mathbf{1}}$ solves (11), i.e. $F^* = F(x_{\mathbf{1}})$. Thus at a Pareto efficient interior allocation, both approximations (12), (13) take value 0, which by corollary 1 is precisely the value of m_x.

[22] Most utilities in the linear risk tolerance class satisfy this, such as CRRA > 1, CARA, log, quadratic.

Approximation (12) partially generalizes formula (6). In fact, with quasilinear utilities $m_x = D_x := F^* - F(x_{\mathbf{1}})$; while, outside the quasilinear case, we can only establish that m_x is bounded above by D_x.

A seeming weakness of theorem 3 is the high level hypothesis on the optimal arbitrage. Restricting to optimal arbitrage of economies in which every agent exerts a nonnegative willingness to pay is equivalent to focus on initial allocations which can be Pareto improved by a simple reallocation of future resources. Indeed, assume that preferences are continuous and increasing in current income; consider an allocation x and a reallocation of future resources z; then it is easily seen that $x^h + (0, z^h) \succeq_h x^h$ if and only if $w^h \geq 0$, for all h.

How sharp are the three approximations of inefficiency? In general,

Proposition 3 *Suppose as in theorems 1, 2, 3. Then the linear, discrete, and quadratic approximations of inefficiency are increasingly sharper:*

$$m_x \leq Q_x \leq D_x \leq L_x$$

The sharpening are dramatic in the economy of section 3.2 with a CRRA parameter of 2.5. As fractions of total current consumption $r_0 = \Sigma x_0^h$, they are reported in Figure 3. The quadratic is very good, the discrete is good, the linear preposterous. Approximations miss the actual inefficiencies, respectively, by 20.5%, 42.6%, 311%.

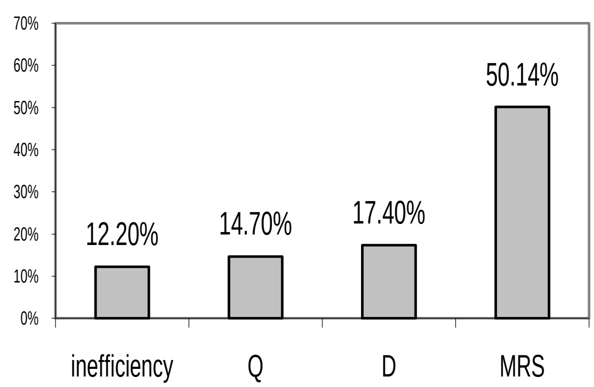

Figure 3: Inefficiency and approximate inefficiency

Notice that the quadratic approximation could be better if the individual in quintile 1 had a nonnegative willingness to pay for the optimal arbitrage. Moreover, in general, for high enough total risk tolerance $\overline{T}_0$, one can conclude the quadratic is only marginally better than the discrete one; for then a first order Taylor expansion shows $Q \approx \frac{\overline{T}_0 D}{\sqrt{\overline{T}_0^2 + 2\overline{T}_0 \cdot D}} \approx \frac{\overline{T}_0 D}{\sqrt{\overline{T}_0^2 + 0}} = D$. But for practical purposes, this conclusion entails computing $\overline{T}_0$, and then one may as well compute the quadratic.

4.2.1 Computing sharper approximations with CRRA felicities

Approximations (12), (13) are explicit up to F^*, which is easy to compute in the following benchmark.

Proposition 4 *Suppose households' future utilities* $u^h_{\mathbf{1}} = \delta^h V$ *differ only in the patience parameters* $\delta^h > 0$, *having the same von Neumann-Morgenstern transform* $V := \Sigma \pi_s v(c_s)$ *of the felicity* $v(c) = \frac{1}{1-\beta} c^{1-\beta}$ *with CRRA* $\beta > 1$. *Then the value of problem (11) is*

$$F^* = \overline{\delta} V(r_{\mathbf{1}})$$

where $\overline{\delta} := \left[\Sigma \left(\mu^h \delta^h \right)^{\frac{1}{\beta}} \right]^{\beta}$ *and* $r_{\mathbf{1}}$ *are total future resources.*

Substituting this in expression (13) with $\mu^h = \frac{1}{u_0^{h\prime}}$ then gives a closed formula for approximate inefficiency. Further, in the quadratic approximation it is easy to compute $\overline{T}_0 = \frac{r_0}{\beta}$ if $u_0^h = v$.

In contrast, a closed formula for inefficiency is hopeless, even in this simplified setting. The equation defining willingness to pay is $\frac{1}{1-\beta}(x_0 - w)^{1-\beta} + \delta \Sigma \pi_s \frac{1}{1-\beta}(x_s + z_s)^{1-\beta} = \overline{u}$, the status quo utility. Solving for w shows inefficiency is hopeless indeed:

$$\sup_{x_{\mathbf{1}} + z \gg 0, \Sigma z^h = 0} \Sigma \left[(1-\beta)\overline{u}^h - \delta^h \Sigma \pi_s \left(x_s^h + z_s^h \right)^{1-\beta} \right]^{\frac{1}{1-\beta}}$$

5 Extensions

We now provide two important extensions to our basic framework and discuss how our main results hold in these new settings. The fist extension is to allow for multiple contingent goods, $L > 1$. This is relevant since most contributions on the existence of inefficiency of equilibria with incomplete asset markets rely on the existence of pecuniary externalities. The second extension is to include production into the economy; this is relevant since it allows to measure the ability of the economy to hedge risk throughout an intertemporal redistribution of resources, as a distinct feature with respect to its ability to hedge risk throughout a redistribution of resources across individuals at each point in time.

5.1 Multiple goods

There is a simple extension of willingness to pay and inefficiency to the case of $L > 1$ commodities per state. It sacrifices generality slightly for simplicity. Thus suppose that preferences admit utility representations that are time separable, $u_0(x_0) + u_{\mathbf{1}}(x_{\mathbf{1}})$, where $(x_0, x_{\mathbf{1}})$ in $R_+^{L(1+S)}$, and that $v_0 = v_0(p_0, I_0)$ denotes the indirect utility associated with u_0. Define a pseudo state space as $\{1, ..., S\} \times \{1, ..., L\}$, with $S^* := SL$ future states. Define a pseudo utility on $\mathbb{R}_+^{1+S^*}$ by $\tilde{u}(I_0, y_{\mathbf{1}}) := v_0(p_0, I_0) + u_{\mathbf{1}}(y_{\mathbf{1}})$, given period 0 prices p_0. The **willingness to pay** for a change in future consumption z in $\mathbb{R}^{S^*}$, in terms of current consumption, is by definition the supremum scalar w such that

$$\tilde{u}(I_0^* - w, x_{\mathbf{1}} + z) \geq \tilde{u}(I_0^*, x_{\mathbf{1}}) \tag{14}$$

where $I_0^* := p_0 x_0$ is the income necessary for the status quo current consumption x_0, in analogy to (4). We note that if x_0 is optimal in that $v_0(p_0, I_0^*) = u_0(x_0)$, then the right side of (14) is just the status quo welfare $u_0(x_0) + u_{\mathbf{1}}(x_{\mathbf{1}})$, so $w(0) = 0$ provided u_0 is increasing.

Definition 2 *Given a current spot price $p_0 \in R_{++}^L$, the **inefficiency** of $(\succeq, x)$ is the value m_{x,p_0} of*

$$\sup \Sigma w^h(z^h) \qquad s.t. \qquad x_{\mathbf{1}} + z \gg 0, \Sigma z^h = 0 \tag{15}$$

Proposition 1 extends:

Proposition 5 *Suppose u_0 is continuous and increasing. Suppose $z \in \mathbb{R}^{HS^*}$ is feasible for (15). Then it is a solution if and only if the $(I_0^{*h}, x_{\mathbf{1}}^h) + (-w^h, z^h)$ define a Pareto optimum (with respect to pseudo utilities).*

The lemmata describing the three approximations of willingness to pay extend to economies with multiple goods.

Lemma 2 (linear) *Suppose the preference is reflexive and convex, and its indifference set through* $x \gg 0$ *is differentiable. The willingness to pay for* z *is at most*

$$w^h \leq \nabla^h z^h$$

Lemma 3 (discrete) *Suppose* Du_0 *is strictly positive and* $D^2 u_0$ *negative definite, let* $v_0' := \frac{dv_0'(p_0, I_0)}{dI_0}$ *be marginal utility. Then willingness to pay is at most*

$$w \leq \frac{\Delta}{v_0'}$$

Lemma 4 (quadratic) *Assume* $v_0''' \geq 0 > v_0''$*. If* $\Delta \geq 0$ *then willingness to pay is at most*

$$w \leq -T_0 + \sqrt{T_0^2 + 2T_0 \cdot \frac{\Delta}{v_0'}}$$

where $T_0 := -\frac{v_0'}{v_0''}$.

As before, these approximations are unambiguously ranked:

Proposition 6 *Suppose as in the lemmata. Then these approximations of willingness to pay are increasingly sharper,*

$$\textit{inefficiency} \leq \textit{quadratic} \leq \textit{discrete} \leq \textit{linear}$$

Lastly, the law of diminishing returns also holds, by an identical argument.

That these upper bounds on willingness to pay translate into upper bounds on inefficiency is merely a notational extension, omitted.

5.2 Including production–a three part decomposition of the welfare cost

We recall that m_x is interpretable as a measure of the inefficiency in the *future* distribution of resources for consumption. If there is a technology for transferring resources across time, there is the potential for inefficiency in the *intertemporal* distribution of resources for consumption. This is measured by the following generalization of our decomposition. Suppose that when $k \in \mathbb{R}_+$ units of the current resource are invested, the technology returns ϕk units of the future resource, where

$\phi \in \mathbb{R}^S_+$ captures a state-contingent productivity shock. If x is the status quo consumption, perhaps gotten by individuals already investing in the technology, then the insurer can change future resources by $\phi(\Delta k)$ with $\Sigma z^h = \phi(\Delta k)$, and extract surplus $W_x(z) - \Delta k$, the total willingness to pay for z net of the investment Δk financing z. For any Δk, denote by $m_x^{\Delta k}$ the maximum surplus from any financeable z :

$$m_x^{\phi}(\Delta k) := \sup_z (W_x(z) - \Delta k) \text{ such that } x + (-w, z) \gg 0, \Sigma z^h = \phi(\Delta k) \tag{16}$$

Clearly, $m_x^{\phi}(\Delta k)$ is a measure of the inefficiency in the consumption distribution of $r_{\mathbf{1}} + \phi(\Delta k)$, in the sense of total willingness to pay to remove it. In fact, the investment Δk is not given but chosen by the insurer to maximize the latter measure:

$$m_x^{\phi} := \sup_{\Delta k} m_x^{\phi}(\Delta k) \tag{17}$$

Note, in case the insurer could not activate the technology, $\Delta k = 0$, then problem (16) reduces to the original one in (2) and so $m_x^{\phi}(0) = m_x$. Yet the insurer can activate it, and the additional surplus he can extract, $i_x^{\phi} := m_x^{\phi} - m_x$, is a measure of the inefficiency in the *intertemporal* distribution of resources for consumption. Thus $i_x^{\phi} = 0$ if $\Delta k = 0$ is optimal; there is no intertemporal inefficiency if the optimal choice is not to activate the technology. Finally, the **total cost** is

$$T_x^{\phi} := \sup_{\Delta k, z} (W_x(z) - \Delta k) \text{ such that } x_{\mathbf{1}} + z \gg 0, \mathbb{E}(\Sigma z^h) = \mathbb{E}(\phi)(\Delta k)$$

This is just problem (17) with insurance as well against the productivity shock ϕ. The **cost of macro risk** is the residual, T_x^{ϕ} minus m_x^{ϕ}.

Appendix

Proposition 1

Proof. Necessity by contradiction. Let z be a solution where the $x^h + (-w^h, z^h)$ admit a Pareto superior reallocation $y \in \mathbb{R}^{H(S+1)}_{++}$, so that $\Sigma y_0^h = r_0 - \Sigma w^h$ and $\Sigma y_{\mathbf{1}}^h = \Sigma x_{\mathbf{1}}^h$, and $y^h \succsim^h x^h + (-w^h, z^h)$ without indifference for some i. By (4) and transitivity, $y^h \succsim^h x^h$. Reduce y_0^i to $y_0^i - \varepsilon$ by some $\varepsilon > 0$. By continuity in current consumption, a small enough ε is feasible, in that still $y^i + (-\varepsilon, 0) \succ^i x^i + (-w^i, z^i) \succsim^i x^i$. But now this modified $\tilde{y}_0$ (identical to y_0 but for household i) sums to $r_0 - \Sigma w^h - \varepsilon$. Set $\tilde{y} := (\tilde{y}_0, y_{\mathbf{1}})$ so $\tilde{y}^h \succsim^h x^h$. Set $a := \tilde{y} - x$ so that

$\tilde{y}^h = x^h + (-(-a_0^h), a_{\mathbf{1}}^h) \succsim^h x^h$ and $\Sigma a_{\mathbf{1}}^h = 0$. This shows that $w^h(a_{\mathbf{1}}^h) \geq -a_0^h$ hence $\Sigma w^h(a_{\mathbf{1}}^h) \geq -\Sigma a_0^h = -\left[(r_0 - \Sigma w^h - \varepsilon) - r_0\right] = \Sigma w^h + \varepsilon$, which exceeds $\Sigma w^h = m_x$, the supremum total willingness to pay for some future reallocation z (which $a_{\mathbf{1}}$ is), a contradiction.

Sufficiency by contraposition. Let $\tilde{z}$ be a counterexample to z being a solution. Consider the two allocations $(x_0^h - w^h(\tilde{z}^h), x_{\mathbf{1}}^h + \tilde{z}^h)$, $(x_0^h - w^h(z^h), x_{\mathbf{1}}^h + z^h)$. By the property of wtp in section 2, they are indifferent to x^h, hence to each other by transitivity. By hypothesis, $\Sigma w^h(\tilde{z}^h) > \Sigma w^h(z^h)$, so that the first allocation has lower current resources than, but of course equal future resources to, the second. Thus change the first allocation by taking the aforementioned current slack and distributing it evenly over households; being increasing in current consumption, this makes the (so modified) first allocation preferred to the second one, using the same resources, showing the second one is not Pareto optimal. ■

Corollary 1

Proof. Necessity. By hypothesis, x is Pareto efficient, and $w^h(0) = 0$ in any case, so $(x_0^h - w^h(0), x_{\mathbf{1}}^h) = x^h$ is Pareto efficient. By proposition 1, $z = 0$ solves problem (5). So the problem's value is $m_x = \Sigma w^h(0) = \Sigma 0 = 0$.

Sufficiency. Suppose $y \in \mathbb{R}_{++}^{H(S+1)}, \Sigma y^h = \Sigma x^h$ with $y^h \succsim^h x^h$. Rewrite as $y^h = x^h + (-(x_0^h - y_0^h), z^h) \succsim^h x^h$, defining $z := y_{\mathbf{1}} - x_{\mathbf{1}}$. Then $w^h(z^h) \geq x_0^h - y_0^h$ by definition of willingness to pay, so $\Sigma w^h(z^h) \geq \Sigma(x_0^h - y_0^h) = 0$. Conversely, $\Sigma w^h(z^h) \leq m_x$ because z is feasible for problem (5) and m_x its value. So if $m_x = 0$, then $\Sigma w^h(z^h) = 0$. The $a^h := x^h + (-w^h(z^h), z^h)$ satisfy $\Sigma a^h = \Sigma x^h$, and are Pareto efficient; this is because at the wtp $w^h = w^h(z^h)$, $x^h + (-w^h, z^h)$ is indifferent to x^h. Thus x is itself Pareto efficient. ■

Lemma 1, Theorem 1

The result relies on a simple **global-infinitesimal principle** for smooth convex preferences: if $x + (z_0, z) \succeq x$ then $(1, \nabla(x)) \cdot (z_0, z) \geq 0$.

Proof of lemma 1. Let w be the willingness to pay for z, so that $x + (-w, z) \sim x$. Since also $x \sim x$, convexity implies $x + t(-w, z) \succsim x$ with $* = tz$ for, say, $t = \frac{1}{2}$. By the global-infinitesimal principle, $(1, \nabla(x)) \cdot (-w, z) \geq 0$, i.e. $w \leq \nabla(x)) \cdot z$. ■

Proof of theorem 1. By principle 1 in section 4, it suffices to show the value of

$$\sup_{x_{\mathbf{1}} + z \gg 0, \Sigma z^h = 0} \Sigma \nabla^h z^h$$

is $\Sigma\left(\bigtriangledown^* - \bigtriangledown^h\right) x_{\mathbf{1}}^h$. Now, $\Sigma z^h = 0$ implies $\Sigma \nabla^h z^h = \Sigma\left(\bigtriangledown^* - \bigtriangledown^h\right)\left(-z^h\right) = *$. In turn, $x_{\mathbf{1}} + z \geq 0$ and $\bigtriangledown^* - \bigtriangledown^h \geq 0$ imply $* \leq \Sigma\left(\bigtriangledown^* - \bigtriangledown^h\right) x_{\mathbf{1}}^h$. So the value is at most the claimed one, which is actually achieved by $z := -x_{\mathbf{1}}$. ■

Proposition 2

Proof. $\boxed{w(z(t)) \geq tw(z)}$ The willingness to pay $w = w(z)$ makes (4) hold with indifference: w solves $x + (-w, z) \sim x$. Of course, $x + (-0, 0) \sim x$. By convexity of the preference, the t-convex combination of the latter left sides is weakly preferred to $x : x + (-tw, z(t)) \succsim x$. Since $w(z(t))$, the willingness to pay for $z(t)$, is the supremum s such that $x + (-s, z(t)) \succsim x$, it follows $w(z(t)) \geq tw$.

$\boxed{\text{Concavity}}$ Fix $s, t \in [0,1]$ and $a \in [0,1]$; we want $w(z(as + a't)) \geq aw(z(s)) + a'w(z(t))$, where $a' := 1 - a$. The following indifferences hold: $x + (-w(z(s)), z(s)) \sim x, x + (-w(z(t)), z(t)) \sim x$. By convexity of the preference, $x + (-aw(z(s)) - a'w(z(t)), *) \succsim x$ where $* = az(s) + a'z(t) = z(as + a't)$. As above, by definition of wtp as the supremum, $w(*) \geq aw(z(s)) + a'w(z(t))$. ■

Theorem 2

We analyze the equation which characterizes the willingness to pay w for a change z in future consumption:

$$u_0(x_0) - u_0(x_0 - w) = \Delta \tag{18}$$

Note, $\Delta, w = 0$ satisfy this equation; since u_0 is increasing, one is positive (negative) if and only if the other is, hence

Remark 1 *The signs of w, Δ agree.*

Lemma 5 (discrete approximation of willingness to pay) *Suppose $u_0' > 0 \geq u_0''$. Then willingness to pay is at most*[23]

$$w \leq \frac{\Delta}{u_0'} \tag{19}$$

[23]To ease notation we omit the argument when it is current consumption x_0 at the status quo, as in u_0' for $u_0'(x_0)$.

Proof. By the Fundamental Theorem of Calculus, $u_0(x_0) - u_0(x_0 - w) = w\int_0^1 u_0'(x_0 - w + tw)dt$. This and equation (18) imply

$$w = \frac{\Delta}{\int_0^1 u_0'(x_0 - w + tw)dt} \tag{20}$$

Since $u_0'' \leq 0$, the integrand is bounded as $u_0'(x_0) \lesseqqgtr u_0'(x_0 - w + tw)$ according as $w \gtreqqless 0$[24], i.e. according as $\Delta \gtreqqless 0$ (by remark 1), giving (19). ■

Proof of theorem 2. By principle 1 in section 4, and upper bound (19), it suffices to show $\sup_{\mathbf{x_1}+z\gg 0, \Sigma z^h = 0} \Sigma \frac{\Delta^h}{u_0^{h\prime}}$ has value $F^* - F(\mathbf{x_1})$. Now,

$$\Sigma\frac{\Delta^h}{u_0^{h\prime}} = \Sigma\mu^h \left[u_{\mathbf{1}}^h(x_{\mathbf{1}}^h + z^h) - u_{\mathbf{1}}^h(x_{\mathbf{1}}^h)\right] = \Sigma\mu^h u_{\mathbf{1}}^h(x_{\mathbf{1}}^h + z^h) - \Sigma\mu^h u_{\mathbf{1}}^h(x_{\mathbf{1}}^h) = F(\mathbf{x_1} + z) - F(\mathbf{x_1})$$

On changing variables as $\mathbf{y_1} = \mathbf{x_1} + z$ and recalling the definition of F^*, this is shown. ■

Theorem 3

Lemma 6 (quadratic approximation of willingness to pay) *Suppose* $u_0''' \geq 0 > u_0'', -u_0'$.[25] *If* $\Delta \geq 0$ *then willingness to pay is at most*

$$w \leq \sqrt{T_0^2 + 2T_0 \cdot \frac{\Delta}{u_0'}} - T_0 \tag{21}$$

Proof. Rewrite expression (20) as $\frac{\Delta}{w} = \int$. (If $w = 0$, then $\Delta = 0$ by remark 1 and the inequality is trivial.) By the Fundamental Theorem of Calculus, $u_0'(x_0 - tw) = u_0' - w\int_0^t u_0''(x_0 - sw)ds$, so this integral is expressible as

$$\begin{aligned}\int &= \int_0^1 u_0'(x_0 - w + tw)dt = \int_0^1 u_0'(x_0 - tw)dt = \int_0^1 \left[u_0' - w\int_0^t u_0''(x_0 - sw)ds\right]dt \\ &= u_0' - w\int_0^1\int_0^t u_0''(x_0 - sw)dsdt = u_0' - w\int_0^1 (1-t)u_0''(x_0 - tw)dt\end{aligned}$$

the latter being the identity $\int_0^1\int_0^t f(s)dsdt = \int_0^1 (1-t)f(t)dt$. Since $u_0''' \geq 0$, in the last integrand we have $u_0'' \gtreqqless u_0''(x_0 - tw)$ for all $t \in [0,1]$ according as $w \gtreqqless 0$.

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[24] That is, $u_0'(x_0) \leq u_0'(x_0 - w + tw)$ if $w \geq 0$; and $u_0'(x_0) \geq u_0'(x_0 - w + tw)$ if $w \leq 0$.

[25] Most utilities in the linear risk tolerance class satisfy this, such as CRRA > 1, CARA, log, quadratic.

Thus,

$$\int \geq u_0' - w\int_0^1 (1-t)u_0'' dt = u_0' - w\frac{u_0''}{2}$$

regardless of $w's$ sign. This and $\frac{\Delta}{w} = \int$ imply $\frac{\Delta}{w} \geq u_0' - w\frac{u_0''}{2}$. Dividing by $-u_0'' > 0$ and using the identity $-\frac{\Delta}{u_0''} = T_0\frac{\Delta}{u_0'}$ imply

$$0 \geq T_0 + \frac{w}{2} - T_0\frac{\Delta}{wu_0'} \tag{22}$$

Finally, suppose $\Delta > 0$. Then multiplying (22) by w, which is positive by remark 1, implies the quadratic $0 \geq \frac{w^2}{2} + T_0 w - T_0\frac{\Delta}{u_0'}$. This being convex, w lies between the roots, hence is at most the greater root, which is $-T_0 + \sqrt{T_0^2 + 2T_0\frac{\Delta}{u_0'}}$.
■

Proof of theorem 3. By principle 1 in section 4, and upper bound (21), it suffices to show

$$\sup_{x_1+z\gg 0, \Sigma z^h=0} \Sigma\left[-T_0^h + \sqrt{T_0^{h2} + 2T_0^h \cdot D^h}\right] \tag{23}$$

has value at most the Q_x in (13), with $D^h := \frac{\Delta^h}{u_0^{h\prime}}$. This is increasing in the $D^h > 0$ ($\Leftrightarrow w^h > 0$, by remark 1). The proof of theorem 2 shows that $\sup_{x_1+z\gg 0,\Sigma z^h=0}\Sigma D^h$ equals $D_x = F^* - F(x_1)$. Thus (23) is at most

$$\sup_{\Sigma D^h \leq D_x} \Sigma\left[-T_0^h + \sqrt{T_0^{h2} + 2T_0^h \cdot D^h}\right]$$

Note, $\Sigma D^h \leq D_x$ will be binding, as the objective is monotone in the $D^h > 0$. Since the objective is concave and the constraint linear, the constraint qualification holds, and Kuhn-Tucker multipliers exist. The FOC are $\frac{T_0^h}{\sqrt{\cdot}} = \lambda$. Rearranging, $\frac{T_0^{2h}}{\lambda^2} = \sqrt{\cdot}^2 = T_0^{h2} + 2T_0^h \cdot D^h$ or $(\frac{1}{\lambda^2} - 1)T_0^h = 2 \cdot D^h$, which aggregated implies $(\frac{1}{\lambda^2} - 1) = \frac{2D_x}{\overline{T}_0}$ so that $D^h = D_x\frac{T_0^h}{\overline{T}_0}$. The insides of the square roots become

$$T_0^{h2} + 2T_0^h \cdot D_x\frac{T_0^h}{\overline{T}_0} = T_0^{h2}\left(1 + \frac{2D_x}{\overline{T}_0}\right)$$

so the objective becomes

$$\Sigma\left[-T_0^h + T_0^h\sqrt{1 + \frac{2D_x}{\overline{T}_0}}\right] = \left(-1 + \sqrt{1 + \frac{2D_x}{\overline{T}_0}}\right)\Sigma T_0^h = -\overline{T}_0 + \sqrt{\overline{T}_0^2 + 2\overline{T}_0 D_x}$$

■

Proposition 3

The proposition relies on a fact: a direction that is globally improving is necessarily locally improving, that tangents lie above the graph of a concave function:

Lemma 7 *Suppose* $f : A \to R$ *is* C^1 *and concave. Then for all* $a+z \in A$, *interior* $a \in A$

$$f(a+z) - f(a) \leq Df(a)z \tag{24}$$

Proof of proposition 3. $\boxed{D_x \leq L_x}$. We recall $D_x = \sup_{x_\mathbf{1}+z \gg 0, \Sigma z^h = 0} \Sigma \frac{\Delta^h}{u_0^{h\prime}}$ and $L_x = \max_{x_\mathbf{1}+z \gg 0, \Sigma z^h = 0} \Sigma \nabla^h z^h$, so it suffices that $\frac{\Delta^h}{u_0^{h\prime}} \leq \nabla^h z^h$. This is immediate from inequality (24), as $\Delta = u_\mathbf{1}(x_\mathbf{1}+z) - u_\mathbf{1}(x_\mathbf{1})$ and $\nabla = \frac{D_{x_\mathbf{1}} u}{u_0^{\prime}}$. $\boxed{Q_x \leq D_x}$. Theorem 3 expresses one in terms of the other, $Q_x = \sqrt{\overline{T}_0^2 + 2\overline{T}_0 \cdot D_x} - \overline{T}_0$. Inequality (24) with $f(D) := \sqrt{\overline{T}_0^2 + 2\overline{T}_0 \cdot D}$ at $D = 0$ gives $Q_x = \sqrt{\overline{T}_0^2 + 2\overline{T}_0 \cdot D_x} - \sqrt{\overline{T}_0^2} \leq \frac{1}{2\sqrt{T_0^2}} 2T_0 \cdot (D_x - 0) = D_x$. ■

Proposition 4

We rewrite the objective $F = \Sigma \mu^h u_\mathbf{1}^h(y_\mathbf{1}^h) = \Sigma \mu^h \delta^h \Sigma \pi_s v(y_s^h) = \Sigma \pi_s m^h v(y_s^h)$ where $m^h := \mu^h \delta^h$. Clearly, $F^* = \Sigma \pi_s a_s$ where

$$a_s := \max_{\Sigma y_s^h = r_s} \Sigma m^h v(y_s^h)$$

The Kuhn-Tucker method leads to the solution $y_s^h = r_s \tau^h$, where $\tau^h = \frac{\left(m^h\right)^{\frac{1}{\beta}}}{M}$ with $M := \Sigma \left(m^h\right)^{\frac{1}{\beta}}$. Thus $a_s = \Sigma m^h v(r_s \tau^h) = k r_s^{1-\beta}$ where $k := \frac{1}{1-\beta} \Sigma m^h \left(\tau^h\right)^{1-\beta}$. This simplifies to $k = \frac{M^\beta}{1-\beta}$, on substituting τ. Thus $a_s = \frac{M^\beta}{1-\beta} r_s^{1-\beta} = M^\beta v(r_s)$ and $F^* = M^\beta \cdot \Sigma \pi_s v(r_s) = M^\beta \cdot V(r_\mathbf{1})$.

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